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particular cases of the following construction with ruler only, whenever a parallel to AB[1] is given: Join O, a point not on AB or the parallel, to A and to B meeting the parallel in D and C respectively. Let DB and CA meet in E, and OE meet AB in F. Join CF meeting DB in G, and let OG meet BA in H, etc. The points F, H, etc., are the same as those found before. This construction was given by Brianchon in 1818 in his *Application de la Théorie des Transversales,* page 37. He remarks: "Ce problème pourrait servir à se former, sur le terrain, une échelle de lever, si on n'avait pas à sa disposition une des mesures reçues, et qu'on connût d'ailleurs la longueur totale de la ligne prise pour échelle."

Lambert gave another construction in his *freye Perspective, oder Anweisung* . . . 1774, pages 173–174.

ARC.

PROBLEMS—SOLUTIONS

19 (Calculus) [1894, 165, 273–275]. Proposed by A. L. FOOTE, Merrick, N. Y.

A and B are in a circular room $2R = 30$ feet in diameter, A being at the center and B at the circumference. B runs around at the rate of $v = 600$ feet per minute and A pursues him at the rate of $u = 100$ feet per minute. How long will the race last, and how far will each have traveled till B is caught?

160 (Calculus) [1902, 271; 1903, 104–106]. Proposed by B. F. FINKEL, Drury College.

A dog at the vertex of a right conical hill pursues a fox at the foot of the hill. How far will the dog run to catch the fox, if the dog runs directly toward the fox at all times and the fox is continually running around the hill at its foot, the velocity of the dog being 6 feet per second, the velocity of the fox being 5 feet per second, the hill being 100 feet high and 200 feet in diameter at the base?

273 (Calculus) [1909, 76, 123–124; 1910, 221]. Proposed by J. SCHEFFER, Hagerstown, Md.

On one side of a circular pond a feet in radius is a duck. On the diametrically opposite side of the pond is a dog. Both swim at the same time, the duck swimming around the circumference of the pond at the rate of m feet a minute, the dog swimming directly towards the duck at the rate of n feet per minute. How far will the dog swim in overtaking the duck?

2801 [1920, 31]. Proposed by A. S. HATHAWAY, Houston, Texas.

A dog at the center of a circular pond makes straight for a duck which is swimming along the edge of the pond. If the rate of swimming of the dog is to the rate of swimming of the duck as $n : 1$, determine the equation of the curve of pursuit and the distance the dog swims to catch the duck.

I. REMARKS AND HISTORICAL NOTES BY R. C. ARCHIBALD AND H. P. MANNING, Brown University.

In 1732 Bourger read before the French Academy a memoir "Sur de nouvelles courbes ausquelles on peut donner le nom de Lignes de Poursuite"[2] in which he solved the following problem: "Trouver la courbe de poursuite, c'est-à-dire la courve par laquelle un vaisseau doit en poursuivre un autre qui s'enfuit par une ligne droit, en supposant que les vîtesses des deux vaisseaux soient toûjours dans le même rapport."

Maupertuis gave[3] a briefer solution of this and formulated also the following more general problem in the solution of which he was led to "l'équation de la courbe en secondes différences": "La courbe *CE* étant donnée; trouver la courbe *BM*, telle que ses tangentes *ME*, coupent sur la courbe *CE* des arcs proportionnels aux arcs *BM*?"

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[1] Or AA' bisected at B; with this given Lambert showed, in 1774, that a parallel to AB can with ruler alone, readily be drawn through any point.

[2] *Histoire de l'Académie Royale des Sciences,* 1732. Paris, 1735, Memoires, pp. 1–14.

[3] Idem, pp. 15–16.

The earliest reference, we have found, to the consideration of the curve *CE* as the arc of a circle is in J. Ficklin's problem in an anonymous article of *The Mathematical Monthly* (Runkle), volume 1 (1859), p. 249. In the problem the velocities of pursued and pursuer are supposed equal. For this case it is stated that "the curve of pursuit continually approaches the circle, to which it becomes an asymptote and meets after an infinite number of revolutions, when the two bodies will be together." In the course of the article (pp. 249–251) other velocity ratios $(m : n) \lesseqgtr 1$ are considered. For $m < n$, the pursuer will never overtake the pursued; "but its nearest approach is a point to be determined." And for $m > n$, the pursuer will overtake the pursued and if the pursuer's motion continues, his "path outside of the circumference will be a wave-like curve, the oscillations growing smaller as the pursuer's distance from the circumference becomes greater." No analyses or reasons are given in support of any of these statements.

The problem of determining the curve of pursuit in the case of the circle interested H. Brocard for several years before he proposed it for solution in *Nouvelle Correspondance Mathématique*, May, 1877, vol. 3, p. 175. No solution being forthcoming he asked for the differential equation of the curve in *Mathesis*, December, 1883, vol. 3, p. 232; this was given by Keelhoff in *Mathesis*, 1886, vol. 6, p. 135.

The problem appears in the following form in *Revue de Mathématiques Spéciales*, February, 1894, vol. 2, p. 272: "Un cheval se meut sur une piste circulaire avec une vitesse uniforme; un jockey parti du centre se dirige continuellement vers le cheval pour l'atteindre, animé lui-même d'une vitesse constante. Quel est la courbe décrite par le jockey?" No answer to this problem has been given in the *Revue*.

Another unanswered query was published in *L'Intermédiaire des Mathématiciens*, October, 1894, vol. 1, p. 183: "En supposant que la lumière d'une étoile mettre douze heurs a parvenir au centre du circle que l'étoile parcourt en vingt-quatre heurs, quelle serait l'équation de la trajectoire décrite par un oiseau qui, partant du centre, se dirigerait constamment, avec une vitesse donnée, vers le point où il voit l'étoile?"

And again in the first volume of this MONTHLY, August, 1894, 273–275 four contributors found for the problem enunciated above by A. L. Foote, that the solution of the differential equation "transcends," as one of the contributors remarks, "the present limits of mathematical genius." A slight variant of this problem appeared fifteen years later in J. Scheffer's problem.

Professor Finkel's form was first published in the *Educational Times*, 1888, problem 9448. "Solutions" were first given in the issue for July 1, 1903; *Mathematical Questions and Solutions*, new series, vol. 5, 1904, pp. 30–31. Professor Finkel made clear the equivalence of his problem with that of the old problem as follows: "Conceive the surface of the cone to be spread out on a plane. . . . This surface may be repeated a sufficient number of times to complete the race. We may conceive the surface as being an infinitely thin membrane and allowed to overlap; then when the race has been continued on the second round about the vertex we may conceive the dog in the surface beneath, and so on for any number of rounds. . . ." The concluding sentence is: "This differential equation has never been integrated so far as I know."

The differential equation of the curve of pursuit for a circle was discussed in print for the first time in an extensive paper by L. Dunoyer in *Nouvelle Annales de Mathématiques*, May, 1906, vol. 65, pp. 193–222. He considered the relative motion of two points, the one moving around the circle and the other starting anywhere in its plane, and showed that the problem led to a differential equation of the form $dx/X = dy/Y$; each of the functions X and Y is a polynomial of the third degree in x and y. The form of the integral curve in the vicinity of a singular point is studied according to principles of Poincaré set forth by Picard in his *Traité d'Analyse*, vol. 3, second edition, 1908, chapter 2.

Mr. F. V. Morley's graphical solution of the problem proposed by Professor Hathaway appears elsewhere in this issue of the MONTHLY (pages 54–61).

The only case when there is any difficulty in deciding whether or not the dog, starting from the center of the pond, will catch the duck is the case not considered by either Mr. Morley or Mr. Dunoyer, namely, when the dog and duck travel at the same rate. This case seems to have been solved by Professor Hathaway alone, see below, unless the anonymous author of the article in Runkle's *Mathematical Monthly*, referred to above, had proved the statement that he made.

To the case of problem 2801 when $n = 1$ an anonymous five-page pamphlet (about $3\frac{1}{4}$ x $6\frac{1}{4}$ inches), without date, has been devoted. It is entitled *A Common Sense Solution of a Curve of*

Pursuit Problem that has been considered unsolvable by many eminent mathematicians.[1] The author is L. T. Houghton of Worcester, Mass.

Mr. Houghton has a relativity theory for his dog and duck problem. He says we might think of the duck as stationary and the pond as revolving in the opposite direction[2]; that it makes no difference in the dog's path through the water whether the pond revolves or the duck swims. If the duck is stationary and the dog moves along the radius to the duck, his path in the water will be carried around with the pond and will be curved. It is necessary for him to swim in the water in a direction oblique to the radius in order to overcome the current as well as to proceed along this radius.

It is the dog's path through the water that Mr. Houghton takes for his actual path when the pond is stationary and the duck swims. This path is characterized by the fact that the dog and duck are always on the same radial line. It is a circle of radius one-half of the radius of the pond, tangent at the center of the pond to the radius which passes through the starting position of the duck.[3] We can prove this by forming the equation of the dog's path, but it may be noted at once that the dog swimming on this circle at the same rate as the duck will always be on a line between the center of the pond and the duck, two positions of the line forming an angle which is at the center of the pond and inscribed in the circle. This makes the dog catch the duck when the latter has swum through an arc of 90°, instead of an arc equal to the diameter as Mr. Houghton supposes.

On this path, however, the dog does not swim directly towards the duck, for its tangent always points ahead of the duck. Mr. Houghton objects strongly to the "tangent method for tracing a pursuer's path," but this seems to be a question as to the meaning of "directly towards." We have simply two problems, Mr. Houghton's problem, where the pursuer is always on a straight line drawn from his original position to the pursued; and the "tangent problem," which is the problem under discussion, in which the pursuer's path is always tangent to the straight line joining the two. According to Professor Hathaway's solution, given below, the tangent problem must be answered in the negative.

II. SOLUTION OF PROBLEM 2801 BY THE PROPOSER.

We shall show that: *When the dog starts from any point in the pond with equal or greater speed, he is invariably drawn into the outline of a leaf of a four-leaved clover fixed on the shoulder of the duck, remaining there and getting as near to the duck as one pleases to name, or eventually catching it.*

Let Q, P (fig. 1) be any positions of dog and duck. The tangents QP at Q and PT at P make angles ψ, ϕ, with a fixed line, and if C be the center and $\theta = \angle CPQ$, we have

$$\psi = \phi + \theta - \frac{\pi}{2}, \qquad d\psi = d\phi + d\theta. \tag{1}$$

If the arcs described from fixed points be S and s, we also have $dS = k\,ds$, where k is the ratio of speeds. Further, $CP = a$, $QP = r$, and, drawing TR perpendicular to QP at R, we have the differential triangle PRT whose sides are

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[1] The following sentences taken from different parts of the pamphlet represent fairly well the position of the author:

"This curve of pursuit problem has estranged old friends and vexed eminent mathematicians. Many wagers have been made which professors of mathematics have been called on to settle, and their decisions have been in the negative [that is, the dog will not catch the duck], without one single line of proof to sustain their findings. A bare assertion is not satisfactory proof." . . .

"The 'nevers' have claimed that the pursuer's position [path] is always tangent to a straight line drawn to the corresponding position of the pursued. This is the modern theory and also a false one. Tracing the dog's path by setting off the circumference into short spaces, and setting off equal distances on the line from the dog to the duck [in its successive positions] will show that the pursuer is always on a straight line drawn from his original position to the pursued, and that the duck is caught when it has moved over a portion of the circumference equal to the diameter."

[2] This idea is virtually the same as the idea, which Professor Hathaway uses (in his solution of problem 2801), of a system of polar coördinates moving with the duck.

[3] See the "turn-table" problem, *Math. Visitor*, 1878, vol. 1, p. 37; also *Math. Quests. Educ. T.*, 1889, vol. 51, p. 157.

$$PT = ds, \qquad PR = dr + dS = dr + kds, \qquad RT = rd\psi = rd\phi + rd\theta.\text{[1]} \tag{2}$$

Since $\angle\, PTR = \angle\, CPQ = \theta$ (sides perpendicular),

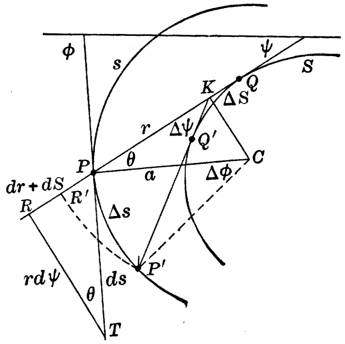

FIG. 1.

$$\sin\theta = \frac{dr}{ds} + k, \qquad \text{or} \qquad \frac{dr}{ds} = \sin\theta - k; \tag{3}$$

$$\cos\theta = r\frac{d\phi}{ds} + r\frac{d\theta}{ds} = \frac{r}{a} + r\frac{d\theta}{ds}, \qquad \text{or} \qquad \frac{d\theta}{ds} = \frac{a\cos\theta - r}{ar} \tag{4}$$

($\Delta s = a\Delta\phi$, therefore $ds = ad\phi$).

Eliminating ds we have

$$r\frac{d\theta}{dr} = -\frac{a\cos\theta - r}{ka - a\sin\theta}. \tag{5}$$

THEOREM 1. *If $k \geqq 1$, r decreases without a minimum; if $k < 1$, $\sin\theta = k$ determines a minimum or maximum value of r.*

Since ds is always positive (s always increasing), dr is always negative by (3) if $k \geqq 1$, but may be positive or negative if $k < 1$. Here $dr = 0$ is a max-min condition, but is not sufficient, namely, dr must change sign. This is not satisfied when $r = 1$, $\sin\theta = 1$. Therefore when $k \geqq 1$ r is always decreasing and has no minimum.

THEOREM 2. *The angle $\theta = \angle\, CPQ$ is increasing or decreasing according as Q is inside or outside of the circle on CP as diameter.*

Draw CK perpendicular to PQ at K. Then, if we take segments along PQ as positive in the direction from P to Q, we have $QK = a\cos\theta - r$, positive or negative, and therefore $d\theta$ positive or negative, according as Q is inside or outside of the circle CKP (r is always positive during the time of the present problem. Only after $r = 0$ may it be negative).

THEOREM 3. *QK is increasing or decreasing according as Q lies outside or inside of the leaves in the first and third quadrants of the four-leaved clover whose equation referred to pole P and axis PC is $kr = a\cos\theta\sin\theta$.*[2]

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[1] We follow Newton's definition. "Differentials are corresponding limits of equimultiples of vanishing differences." Also, "instantaneous state" is mathematically defined as that variation in which differentials are increments, just as in actual variation differences are increments. It is demonstrated, not assumed, that if a point be generating a curve, its instantaneous state generates the tangent, etc. Taking differences from a succeeding position, $Q'P' = r + \Delta r$ (Fig. 1), unwrap an inextensible string $QQ'P'$, from the arc QQ', describing the arc $P'R'$ always normal to the string.

$$\text{arc } PP' = \Delta s, \qquad PR' = \Delta r + \Delta S, \qquad \text{arc } R'P' = q\Delta\psi, \qquad r + \Delta r < q < r + \Delta r + \Delta S.$$

Extend PR' and PP' in the ratio $N : 1$ so large that $N \cdot PP'$ is as near as we please to PT, when $Q'P'$ is as near as we please to QP. The differences and figure $PR'P'$ are vanishing, but the finite figure, similar to $PR'P'$, approaches a limit, namely, the right triangle PRT. Hence the above differential values, by definition of differentials. See *Science*, July 11, 1919, and Feb. 13, Mar. 28, May 7, June 11, July 9, 1920, for correspondence on the early history and concepts of the calculus.

[2] This curve, so named by M. Simon (*Analytische Geometrie*, Leipzig, 1900, p. 316), and called also the four-leaved rose curve, and the quadrifolium, is one of the class of rose curves or rhodonees, $r = a\sin m\theta$, $r = a\cos m\theta$, discussed by Guido Grandi in letters to Leibnitz, 1713, in *Philosophical Transactions*, 1723, and in his book, *Flores geometrici ex rhodonearum et clœliarum curvarum descriptione resultantes*, 1728. These curves have been also called corollae by W. J. C. Miller (*Math. Questions with their Solutions*, London, vol. 19, 1873, p. 60) on account of their fanciful resemblance to the petals of an open flower. It was remarked empirically in 1752, by G. B. Suardi, that the corollae can all be generated as elongated or contracted epi- or hypocycloids, but the proof of this fact was first given in 1844 by L. Ridolfi Bellavitis showed in 1852 that the pedal curve of any ordinary epi- or hypocycloid with respect to the center of its fixed circle is a rose curve; in particular the four-leaved clover is the pedal curve of the so-called four-cusped hypocycloid or astroid. It is also the orthoptic curve of the astroid (Lambiotte, 1877) and the inverse of the

These leaves are fixed on the duck, since the axis PC is so fixed. Also with that axis and pole (r, θ) represents Q. Then

$$\frac{d(QK)}{ds} = -a \sin\theta \frac{d\theta}{ds} - \frac{dr}{ds} = \frac{kr - a\cos\theta\sin\theta}{r}, \qquad [(3), (4)]$$

and $d(QK)$ is positive or negative with $kr - a\cos\theta\sin\theta$. Since r is positive, the latter is positive, not zero, in the second and fourth quadrants, and is zero only on the leaves in the first and third quadrants, positive outside, negative inside. This problem excludes the second and third quadrants, but in generalizing the start to any point on land, with the same ratio of velocities, approach may be from any quadrant.

Theorem 4. *When $k \geqq 1$, the pursuing point Q can cross the clover leaf of the first quadrant only from outside to inside.*

Put $u = kr - a\cos\theta\sin\theta$, then by (3) and (4) we have, when $u = 0$ (Q on the clover leaf).

$$\frac{du}{ds} = \frac{\sin\theta - k}{\sin\theta} |\cos^2\theta + \sin\theta(k - \sin\theta)| < 0$$

since $\sin\theta > 0$. Therefore u is decreasing, and changes from positive to negative, or Q crosses from outside to inside.

If $k < 1$, then on the first quadrant leaf, entrances are up on the arc from P to $\sin\theta = \frac{1}{4}(k + \sqrt{k^2 + 8})$, and down from $\sin\theta = k$ to P; the arc between these rays contains exits only.[1]

Theorem 5. *If $k \geqq 1$ the first quadrant clover leaf lies wholly inside of the semicircle CKP; if $k < 1$ the leaf intersects the semicircle on the chord $\sin\theta = k$.*

Theorem 6. *The angle $CPQ = \theta$, and the distance $CQ = z$ are both increasing or both decreasing.*

From the triangle CPQ, $z^2 = a^2 + r^2 - 2ar\cos\theta$,

$$z\,dz = (r - a\cos\theta)dr + ar\sin\theta d\theta = kar\,d\theta$$

by (5); therefore

$$\frac{dz}{d\theta} = \frac{kar}{z} > 0.$$

The presence of Q in the circle CKP, if $k \geqq 1$, can now be shown, wherever it starts in the pond. When $k > 1$ we can say $dr/ds \leqq 1 - k$, a fixed number less than zero, and that r becomes zero for a finite value of s. When $k = 1$ we can say by theorem 2 that θ is always decreasing outside of the circle CKP. Suppose we start with the value θ_1, Q within the pond, so that $z = c < a$. As long as Q is outside of this circle θ will decrease, but will not decrease as far as $-\pi/2$, for z will also be decreasing. Then we have $dr/ds \leqq \sin\theta_1 - 1$, and therefore for a finite value of s, r becomes zero *or* Q comes into the circle. But in any case before r becomes zero z will be greater than c, and must sometime be increasing; then θ will be increasing, and Q inside of the circle CKP (theorem 2). Hereafter we start from this circle, dismissing $k < 1$ with the remark that r is increasing in the angle between the two rays $\sin\theta = k$ of the first and second quadrants, and decreasing outside of that angle, while θ is increasing inside of the circle CKP, and decreasing outside. There is a minimum r on $\sin\theta = k$ in the second quadrant, and one in the first quadrant inside of the circle CKP, and a maximum r on $\sin\theta = k$ in the first quadrant outside of that circle.

We found (5) to be the differential equation of pursuit as the duck sees it, that is, referred to the axis PC fixed on the duck. Let the plane of pursuit revolve about C with opposite the angular velocity of P (Fig. 2). No relative positions are changed, but an observer on the ground will see only a fixed point P and axis PC; and Q will appear to be moving on the ground, no longer towards P but in a direction QZ whose inclination to the radius vector QP is given by (5).

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circular cross curve (Kreuzcurve)—concerning which considerable has been written since it was first conceived by Terquem in 1847.—Editor.

[1] In the leaf of the third quadrant, for all values of k, entrances are below, exits above, the ray of angle θ given by $\sin\theta = -2/(k + \sqrt{k^2 + 8})$ [180° to 225°], showing a drift round behind the duck.

QZ is constructed as follows: Produce CK to $CM = ka$ (in the positive sense, CK being equal to $a \sin \theta$, so that $KM = ka - a \sin \theta$). Then the direction of QZ is determined by revolving QM through a counter-clockwise right angle. For, the tangent of the angle which QM makes with the radius vector is $\dfrac{ka - a \sin \theta}{a \cos \theta - r}$, which by (5) is the negative reciprocal of the tangent of the angle made by QZ. By continuity, one case is a sufficient test of sense: when Q is on the circle CKP it moves towards P ($k \geqq 1$).

THEOREM 7. *Capture can take place only from directly behind or directly in front.*

If the chord QP of the curved path QP in Fig. 2 be vanishing, its angle with the tangent QZ must also be vanishing, and the equal angle KMQ. Therefore $QK = a \cos \theta - r$ is vanishing, and since $\lim r = 0$, $\lim \cos \theta = 0$, or the limit of θ is a right angle.

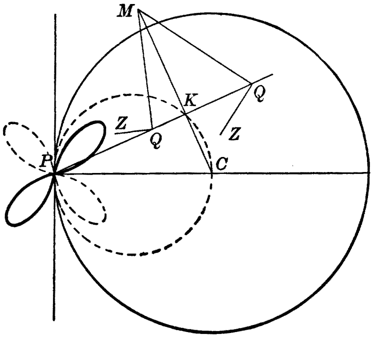

FIG. 2.

In the second and third quadrants PQ turns clockwise and all pursuit starting from there must capture from behind. Also all starting from the pond, or eventually in the circle CKP, must capture from behind. For, in the circle CKP QK is positive, and $\lim QK = 0$ (theorem 7), therefore QK must eventually decrease, and this is only true for Q in the clover leaf of the first quadrant (theorem 3) from which it cannot get out (theorem 4). This leaf is within the circle CKP (theorem 5), and θ must continuously increase to its limit $\pi/2$.

THEOREM 8. *The duck is caught or not caught according as* (*a*) $k > 1$, *or* (*b*) $k = 1$.

Let accents denote corresponding values. By the mean value theorem

$$(r' - r)/(s' - s) = \sin \theta'' - k, \qquad \theta < \theta'' < \theta' \quad (dr/ds = \sin \theta - k).$$

Therefore

$$(a)\ k > 1, \qquad s' - s = (r - r')/(k - \sin \theta'') < r/(k - 1).$$

Since any distance, $s' - s$, that the duck can go without being caught is less than a fixed distance it must be caught.

$$(b)\ k = 1, \qquad s' - s > (r - r')/(1 - \sin \theta).$$

Now if the duck is caught after going a finite distance, so that $\lim r' = 0$, we shall have eventually $s' - s > r/(1 - \sin \theta)$. But

$$\frac{r}{1 - \sin \theta} = \frac{a \cos \theta}{1 - \sin \theta} - \frac{a \cos \theta - r}{1 - \sin \theta} = \frac{a(1 + \sin \theta)}{\cos \theta} - a \tan \angle KMQ,$$

which may be made as large as we please, for we may start as near the duck as we please and take θ as near $\pi/2$ and $\angle KMQ$ as near zero as we please. This leads to a contradiction, and proves that the duck will not be caught in a finite time when $k = 1$.

We can employ the methods of this article when the path of the pursued is any curve, C being the center of curvature, a the radius of curvature, and $d\phi/ds = 1/a$ the curvature, but in differentiations a would not be constant.

The following dynamical development of the preceding is of interest. On the plane of pursuit draw the evolute of the path of the pursued, and let it roll on a fixed line carrying the plane of pursuit with it. When the pursued crosses the fixed line let the rolling motion begin and be always of velocity equal and opposite to that of the pursued about its center of curvature. Then the pursued remains in a fixed position on this line. For the actual motion of P is that of a fixed point on a line which rolls on the evolute, the point of contact C being its center of curvature at the instant. When the line rolls from C to C', bringing P to P', the arc CC' of the evolute rolls on the fixed line CP, C' falling at C'' on CP, and bringing P' back to P. All that varies is the position of the center of curvature from C to C'' on CP.

At the instant when C is the center of curvature and v the velocity of P the latter point has in the rolling plane a counter-clockwise angular velocity v/a about C, and this rolling plane, rolling to keep P stationary, has a clockwise angular velocity v/a about C. The apparent motion of the pursuing point Q is the resultant of this clockwise angular velocity and a linear velocity kv towards P. This resultant is the same angular velocity about a new center M. Hence CM is perpendicular to PQ, and in sense so that, if turned a clockwise right angle about C it will lie in the direction PQ. Also $CM \cdot v/a = kv$, or $CM = ka$. Therefore the apparent velocity of Q is $MQ \cdot v/a$ in the direction QZ found by rotating QM about Q a counterclockwise right angle. From these considerations (3) and (4) may be derived.

NOTES AND NEWS.

It is hoped that readers of the MONTHLY will cooperate in contributing to the general interest of this department by sending items to the Editor-in-Chief.

Mr. F. E. WOOD, who received his doctorate at the University of Chicago in December, has been appointed assistant professor of mathematics at the Michigan Agricultural College.

At Colgate University, Associate Professor A. W. SMITH has been made full professor and head of the department of mathematics as successor to Professor J. M. TAYLOR [*1921*, 43]; Professor T. R. AUDE, of the Carnegie Institute of Technology, has been appointed associate professor of mathematics.

Dr. L. A. POCHHAMMER, ordinary professor of mathematics at the University of Kiel for forty-three years, died on March 24, 1920, at the age of seventy-eight years. He was appointed extraordinary professor at Kiel in 1874, and was the author of a number of papers in *Mathematische Annalen* and Crelle's *Journal.*

At the meeting of the Mathematical Association of America, at the University of Chicago, December 28–29, the following officers were elected: President, Professor G. A. MILLER; Vice-presidents, Professor R. C. ARCHIBALD and Professor R. D. CARMICHAEL; Board of Trustees, to serve till January, 1924, Professor A. A. BENNETT, Professor FLORIAN CAJORI, Professor H. L. RIETZ and Professor D. E. SMITH. The Board chose Professor C. F. GUMMER to fill the vacancy on the Board caused by the election of Professor Carmichael as vice-president.

At the meeting of the American Mathematical Society, at Columbia University on December 28–29, 1920, the following officers were elected: President, Professor G. A. BLISS; Vice-presidents, Professor F. N. COLE and Professor DUNHAM JACKSON; Secretary, Professor R. G. D. RICHARDSON; Treasurer, Professor W. B. FITE; Librarian, Professor R. C. ARCHIBALD; Committee of Publication (to edit the *Bulletin*), Professor E. R. HEDRICK, Professor W. A. HURWITZ, and Professor J. W. YOUNG. The following members of the council were elected to serve until December, 1923: Professor T. H. GRONWALL, Professor O. D. KELLOGG, Professor FLORENCE P. LEWIS, and Professor A. D. PITCHER.

Announcement has been made at Brown University of the completion of the NATHANIEL FRENCH DAVIS FUND in honor of Professor Davis, now emeritus, who was for forty-one years a teacher of mathematics in the University. The Fund amounts to ten thousand dollars and the income is to supplement the regular library appropriations in purchasing mathematical books and periodicals for the mathematical seminary.